2014

[This question paper contains 10 printed pages.]

6135 *Your Roll No.* ....

B.Sc. (H) / Computer Science / VI Sem. **D**

Paper–604

OPERATIONAL RESEARCH TECHNIQUES

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** *questions in all.*
Question No. **1** *is compulsory.*
Use of non-programmable scientific calculators is permitted.

1. (a) Qzark Farms uses at least 800 lb of special feed daily. The special feed is a mixture of Corn & Soybean Meal with the following composition :

Feed Stuff	Protein	Fiber	Cost ($/lb)
Corn	0.08	0.04	0.35
Soyabean Meal	0.65	0.08	0.95

The dietary requirements of the special feed are at least 30% protein and at the most 5% fiber. Ozark farms wishes to determine the daily minimum - cost feed mix. Formulate the above as linear programming problem. (5)

(b) The following tableau represents a specific simplex iteration that is not optimal. All variables are non-negative. Suppose that the problem is of the maximization type. Identify the non-basic variable that has the potential to improve the value of z. If such variable enters the basic solution, determine the associated leaving variable, if any, and the associated change in z.

Basic variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Solution
X8	0	3	0	–2	–3	–1	5	1	12
X3	0	1	1	3	1	0	3	0	6
X1	1	–1	0	0	6	–4	0	0	0
Z coefficient	0	–5	0	4	–1	–10	0	0	620

(5)

(c) In a 3X3 transportation problem, let x_{ij} be the amount shipped from source i to destination j and let c_{ij} be the corresponding transportation cost per unit. The amounts of supplies available at sources

1,2 and 3 are 15, 30 and 85 units respectively and the demands at destinations 1, 2 and 3 are 20, 30 and 80 units respectively. Assume that the starting north-west corner solution is optimal and the associated values of the multipliers are given as $u_1 = -2$, $u_2 = 3$, $u_3 = 5$, $v_1 = 2$, $v_2 = 5$ and $v_3 = 10$. Find the associated optimal cost. (5)

(d) Consider the following LPP

Minimize $z = 4x_1 + x_2$

subject to

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

If the artificial variables x_4, x_5 and the slack variable x_6 are the starting solutions, using M = 100 for artificial variables, the optimal table for primal is

Basic variables	X_1	X_2	X_3	X_4	X_5	X_6	solutions
X_1	1	0	0	0.4	0	–0.2	0.4
X_2	0	1	0	0.2	0	0.6	1.8
X_3	0	0	1	1	–1	1	1
Z coefficient	0	0	0	–98.6	100	–0.2	

Write the associated dual problem. Determine the optimal solution of the dual through the above given table. (6)

(e) For the queuing model M/M/1/N, set up the transition diagram and then write and solve the steady state equations to determine P_n, the probability that there are n customer in the system. (6)

(f) A 4-ton vessel can be loaded with one or more of the three items. The following table gives the unit weight w_i, in tons and the unit revenue in thousands of dollars, r_i, for item i. How should the vessel be loaded to maximize the total return ?

Item i	Weight w_i	Revenue r_i
1	2	31
2	3	47
3	1	14

(8)

2. (a) Solve the following Linear Programming problem graphically

Maximize $z = 2x_1 + x_2$

subject to $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0$

(b) Discuss the applications of Operations research in Computer science. (7,3)

3. (a) Use simplex method to solve the following linear programming problem :

Minimize $z = 5x_1 + 3x_2$

subject to $2x_1 + 4x_2 \leq 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \geq 10$

$x_1, x_2 \geq 0$

(b) Consider the following linear programming problem

Maximize $z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 + x_4 = 30$

$3x_1 + 2x_3 + x_5 = 60$

$x_1 + 4x_2 + x_6 = 20$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

It is given that x_4, x_5, x_6 are the slack variables. Write the dual of this primal problem. Find the values of the optimal dual variables and check the feasibility of the solution when it is given that the optimal primal basic solution is

Basic variables : (x_2, x_3, x_1),

$$\text{Inverse} = \begin{bmatrix} \frac{1}{4} & \frac{-1}{8} & \frac{1}{8} \\ \frac{3}{2} & \frac{-1}{4} & \frac{-3}{4} \\ -1 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(5,5)

4. (a) A product is manufactured at four factories A,B,C and D and is supplied to four stores I, II, III and IV. The unit transportation costs of transportation are given in the following table. Use Vogel's Approximation Method to find the initial basic feasible solution so as to minimize the transportation cost.

Factories	Stores				
	I	II	III	IV	Supply
A	4	6	8	13	50
B	13	11	10	8	70
C	14	4	10	13	30
D	9	11	13	8	50
Demand	25	35	105	20	

(b) An airline company has drawn up a new flight schedule involving five flights. To assist in allocating five pilots to the flights, it has asked

them to state their preference scores by giving each flight a number out of 10. The higher the number, the greater is the preference. Certain of these flights are unsuitable to some pilots owing to domestic reasons. These have been marked with a X. What should be the allocation of the pilots to flights in order to maximize the preferences ?

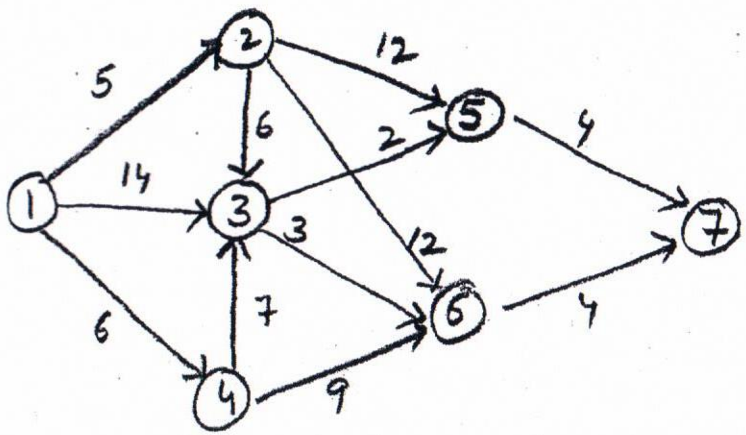
		Flight Number				
Pilots		1	2	3	4	5
	A	8	2	X	5	4
	B	10	9	2	8	4
	C	5	4	9	6	X
	D	3	6	2	8	7
	E	5	6	10	4	3

(5,5)

5. (a) Use graphical method in solving the following game and find the value of the game.

Player A	Player B			
	B_1	B_2	B_3	B_4
A_1	2	2	3	–2
A_2	4	3	2	6

(b) A salesman located in city A decided to travel to city B. He knew the distances of alternative routes from city A to city B. He then drew a highway network map. The city of origin A, is city 1. The destination city B, is city 7. Other cities through which the salesman will have to pass through are numbered 2 to 6. The arrows representing routes between cities and distances in kilometers are indicated on each route. The salesman's problem is to find the shortest route that covers all the selected cities from A to B. (5,5)



6. A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity (i-j)	**Estimated Duration (weeks)**		
	Optimistic **(t_o)**	**Most likely** **(t_m)**	Pessimistic (t_p)
1–2	1	1	7
1–3	1	4	7
1–4	2	2	8
2–5	1	1	1
3–5	2	5	14
4–6	2	5	8
5–6	3	6	15

(i) Draw the project network and find the critical path.

(ii) Find the expected duration and variance of the project.

(iii) What is the probability that the project will be completed 4 weeks earlier than the expected **time ?** (10)

7. **(a) Customers arrive at a** box office window, being **manned by a single individual** according to a **Poisson input process with a mean rate** of 30 per **hour. The time required to serve** a customer has **an exponential** distribution with a mean of 90 **seconds.** Find

(i) Average number of customers in the system

(ii) Average number of customers in the queue

(b) Consider the following quadratic programming problem :

Maximize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to $x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0$

Derive the Kuhn-Tucker conditions for optimality and perform two iterations of Wolfe's method to get an optimal solution. (5,5)